XXI. A Two-level Stochastic Model to Estimate Vessel Throughput Time

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Abstract:
A good estimate of the vessel sojourn time is essential for better planning and scheduling of container terminal resources, such as berth scheduling, quay crane (QC) assignment and scheduling, and fleet size planning. However, estimating the expected vessel sojourn time is a complex exercise because the time is dependent on several terminal operating parameters such as the size of the vessel, the number of containers to be loaded and unloaded, and the throughput of the QCs. The throughput of the QCs in turn depends on the type and number of transport vehicles, number of stack blocks, the topology of the vehicle travel path, the layout of the terminal, and several event uncertainties. To address the modelling complexity, we propose a two-level stochastic model to estimate the expected vessel sojourn time. The higher level model consists of a continuous-time Markov chain (CTMC) that captures the effect of QC assignment and scheduling on vessel sojourn time. The lower level model is a multi-class closed queuing network (CQN) that models the dynamic interactions among the terminal resources and provides an estimate of the transition rate input parameters to the higher level CTMC model. We estimate the expected vessel sojourn times for several container load and unload profiles and discuss the effect of terminal layout parameters on vessel sojourn times.

1 Introduction

The adoption of containers for sea-freight transport offers numerous advantages such as effective handling of cargo, easier storage, reduced costs of transport, and faster transshipment. The world container throughput is estimated to reach 1 billion TEU (20 ft equivalent unit) by 2020 (www.apmterminals.com), which is almost two times the current container traffic. There is a need to improve the operational performance of container terminals to handle large ships in a short time, and at acceptable costs.

The operations at a new-generation automated container terminal can be broadly classified into seaside operations and landside operations. The seaside operations can be further classified into quay crane (QC) operations, vehicle operations, and automated stack crane (ASC) operations. The QC operations begin after allocation of berth space to the incoming vessels. Then the QCs are assigned to the bays of the vessel to unload and load containers. The vehicles transport containers between QC buffer lanes and ASC buffer lanes. The ASCs store inbound containers into the stack buffers and retrieve outbound containers and load them on the vehicles. The landside operations comprise movement of containers between stack buffers and landside trains or trucks. If operations are not properly managed, the turnaround time of ships increases, leading to high costs for the shipping liners and penalties for the terminal operators.

In order to manage the ship turnaround time, and guarantee the ship can be handled within its allotted slot, it is important to be able to estimate the turnaround time, depending on the resources allocated to it. We define the vessel sojourn time as the time taken to fully unload and load containers to a vessel, which is largely dependent on the number of containers to be loaded and unloaded, the number of QCs assigned to a vessel and QC productivity. The vessel sojourn time is particularly impacted if the QCs have to wait for vehicles to bring or take away containers. This waiting time in turn depends on other work at the terminal (other vessels), type of vehicles, number of vehicles, topology of the vehicle travel path, and the layout of the terminal in general. The number of QCs that can be assigned to a vessel for unloading and loading containers depends on the size of the vessel, the safety distance to be maintained between two adjacent QCs, and the number, sequence, and location of the containers on board to be loaded and unloaded.

In literature, the problem of determining the number of QCs to be assigned to a vessel is known as the Quay Crane Assignment Problem (QCAP). The principal objective of all terminal operations is to minimize the duration of time for which a vessel stays at the terminal. The order in which containers are unloaded and loaded by each QC can significantly alter the sojourn time of a vessel. Thus, optimal sequencing of tasks performed by each QC is necessary. This problem is widely known as the Quay Crane Scheduling Problem (QCSP). The earliest work on the QCSP is by Daganzo [1989] and Peterkofsky and Dazango [1990]. They assume one crane per hold is assigned for each vessel. Daganzo [1989] formulates a mixed integer program for the QCSP considering multiple vessels and presents both exact and approximation methods to solve the problem for small instances. Peterkofsky and Dazango [1990] attempt to minimize the delay costs of the vessels and propose a branch and bound method to solve the problem. Kim and Park [2004] and Lim et al. [2004] also address the QCSP by modeling additional operational constraints such as non-interference of QCs, safety distance between adjacent QCs, and precedence relationship among the tasks. Studies in the area of QCSP include those of Bierwirth and Meisel [2010] and Meisel and Bierwirth [2011], where the authors classify various QCSP models, assess their solution methods, and examine the conditions for these solution methods.

There are several sources of uncertainties in the quayside and the stackside operations that add to the variability in the time to discharge (load) containers from (to) the vessel. For instance, the time to unlash the containers on the vessel before discharging is highly variable (typically outsourced to a third-party company), the time to remove the hatch covers and open the twist locks varies, the position of the container in the vessel affects the QC operator's time to position the crane, or a poor stowage plan at the port of origin can increase the number of container restows before the target container can be discharged at the destination port. Other QC factors such as handling non-standard containers (such as 45 ft containers, reefer containers, tank containers), QC break-downs, and differences in skills between the crane crews, add to the discharge time variability. Likewise, there are several sources of uncertainties in the stackside area as well that affect the time to stow or retrieve the containers from the stack blocks, indirectly affecting the vessel sojourn time variability. For instance, the right container may not be immediately available for loading, and reshuffling of containers need to be done. Furthermore, ASC breakdowns may occur, which affect the stack storage and retrieval time variability.

A deterministic model is clearly insufficient to capture the variability in the container discharge and loading operations as it may lead to severe underestimation of the sojourn time (also see Roy and de Koster [2012]). We therefore propose a two-level stochastic model. The higher level model consists of a continuous-time Markov chain (CTMC) with a single absorbing state that marks the end of vessel unloading and loading operations, and the vessel is ready to depart the terminal. The lower level model uses closed-queuing network models to provide the transition rate inputs to the higher level model. Using this model, we first estimate the expected vessel sojourn times and then show the effect of quay and stack crane service time variability on the vessel sojourn times.

The rest of this paper is organized as follows. In Section 2, we describe the arrangement of containers in the vessel and vessel handling process. The analytical model to estimate the vessel sojourn time is explained in Section 3. The results from numerical experiments are shown in Section 4 and the conclusions from this study are drawn in Section 5.

2 Vessel Handling Process

We now discuss the arrangement of containers on the vessel. We only consider 20 ft. standard size containers and assume that all containers are identical in size and shape. Containers are stored on the vessel in bays, tiers, and rows. Bays and rows describe the location of a container across the length and width of the vessel respectively (see Figure 1). Containers are vertically stacked in tiers. Therefore, each container on the vessel is uniquely characterized by its bay, row, and tier number. Further, in our model, we partition the total number of bays into two zones, namely, Z_1 and Z_2, and assign one QC to each zone for unloading and loading containers from/to the vessel.

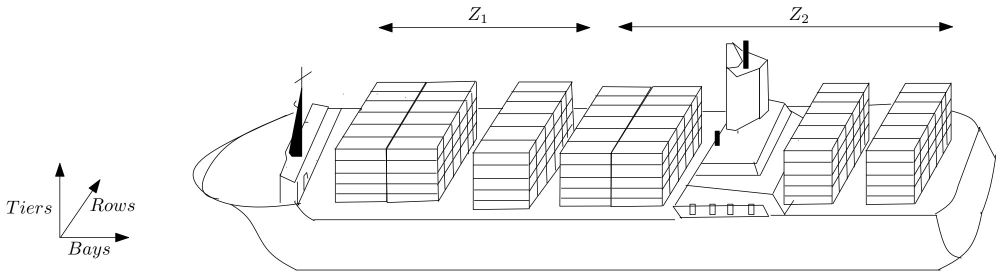

Figure 1: The layout of the container vessel considered in this research

The terminal broadly consists of three areas, namely the quayside, the stackside, and the vehicle transport area. The quayside area has two QCs performing both unloading and loading operations on the vessel. The stackside area constitutes N_s stack blocks, each served by an Automated Stacking Crane (ASC) for storage and retrieval of inbound and outbound containers. The vehicle transport area comprises a unidirectional rectangular travel guide path on which V AGVs transport containers between the quayside and the stackside. The vehicle guidepath contains two shortcut paths along the two QCs to reduce the travel time from quayside to stackside. Automated Guide Vehicles (AGVs) unlike Automated Lift Vehicles (ALVs), cannot lift the container from the ground by themselves. The containers need to be put on the AGVs by both quay and stack cranes. Thus, if the AGV is not present, a crane waits for the vehicle to arrive to pickup or dropoff a container.

QC_i is assigned to zone Z_i for unloading the containers for $i = 1, 2$. A QC unloads the container from the vessel and places it on the AGV stationed at the quayside buffer lane. The AGV travels along the vehicle guide path to one of the stack buffer lanes where the container is lifted from the AGV by the corresponding ASC. The ASC then stores the container in the stacks and the AGV reverts to the quay for next cycle of operation. Similar operations are performed for loading the containers to a vessel but in the reverse sequence. Note that loading of containers to a zone of the vessel begins only after all containers have been unloaded from it.

When all the containers have been unloaded from Z_i, QC_i immediately starts loading containers to it. The QC which first finishes loading the containers to its zone may then choose to either leave the vessel (to attend to another vessel of higher priority) or to assist the other QC (cooperate) to unload/load containers from the adjacent zone. In the latter case, both QCs jointly unload/load containers from one zone. When all containers have been loaded to each zone, the vessel departs. We assume the QC times to unload all containers from a zone or load all containers to a zone to be exponentially distributed.

3 Model Description

The vessel sojourn time depends primarily on the throughput of the QC. However, the throughput of the QC in turn depends on the terminal processes such as the stackside and the vehicle transfer process. To address this complexity, we propose a two-level stochastic model to estimate the expected vessel sojourn time.

The higher level model consists of a continuous-time Markov chain (CTMC) with single absorbing state that captures the effect of QC assignment and scheduling on vessel sojourn time. The lower level model, which is a multi-class closed queuing network (CQN) is developed for each state present in the CTMC. The QC throughput measure from the lower level CQN provides an input to estimate the transition rate parameters present in the higher level CTMC model. The vehicle travel service time parameter for the travel queues captures the effect of the vehicle guide path and the location of the QCs and ASCs. In this way our model captures the effect of terminal layout on the sojourn time. The higher-level model, which is a Continuous-time Markov Chain model, is described in Figure 2. We make the following assumptions in our model.

- The time taken to finish unloading (loading) task on zone Z_i by QC_i is exponentially distributed with rate μ_i (λ_i) for $i = 1, 2$.
- Upon finishing loading to Z_i, QC_i ($i = 1, 2$) may choose to either leave the vessel or assist in unloading/loading the other zone with probabilities described in Table 1.
- The time taken by the QCs to jointly finish unloading (loading) task on Z_i is exponentially distributed with rate α_i (β_i) for $i = 1, 2$.

Table 1: Notations used in the higher level CTMC

U	Unloading of containers from Z_i by QC_i for $i = 1, 2$.
L	Loading of containers to Z_i by QC_i for $i = 1, 2$.
U_j	Joint unloading of containers from Z_i by both QCs for $i = 1, 2$.
L_j	Joint loading of containers to Z_i by both QCs for $i = 1, 2$.
$*$	QC_i leaves the vessel for $i = 1, 2$.
p_1	Probability of QC_1 leaving the vessel after loading Z_1 when QC_2 is still unloading Z_2.
p_2	Probability of QC_2 leaving the vessel after loading Z_2 when QC_1 is still unloading Z_1.
p_3	Probability of QC_1 helping QC_2 to load containers to Z_2 after jointly unloading Z_2.
p_4	Probability of QC_1 leaving the vessel after loading Z_1 when QC_2 is still loading Z_2.
p_5	Probability of QC_2 leaving the vessel after helping QC_1 to jointly unload Z_1.
p_6	Probability of QC_2 leaving the vessel after loading Z_2 when QC_1 is still loading Z_1.

State space of the CTMC:
We define the state of the CTMC as a two-tuple where the i^{th} component represents the task being performed in Z_i at any instant of time for $i = 1, 2$. For instance, (U, U) indicates that containers are being unloaded by both QCs in their respective zones (QC_i in Z_i for $i = 1, 2$). Further, we consider a CTMC with a single absorbing state. The state of absorption corresponds to the state from which no further transitions occur. In our model, the state of absorption represents the state when all containers have been loaded to both Z_1 and Z_2 and the vessel is ready for departure. The state space $\mathscr{S}$ is the set of all possible states of the system.

$\mathscr{S} = \{(U, U)$, (L, U), (U, L), $(*, U_j)$, $(*, U)$, (L, L), $(U, *)$, $(U_j, *)$, $(*, L)$, $(L, *)$, $(*, L_j)$, $(L_j, *)$, $(*, *)\}$, where the different symbols identifying a state are explained in Table 1.

At the beginning of the time period, both QCs start unloading containers from their respective zones. Therefore, the initial state of the system is state 1 i.e., (U, U). We analyze the evolution of the CTMC states over time until it reaches the absorption at state 13 i.e. $(*, *)$. The state transition diagram is shown in Figure 2.

We explain a sample path that the system follows to reach absorption. For instance, let us consider the case where the QC_1 is the first to finish unloading containers from its zone Z_1. This causes the CTMC transition from state 1 to state 2 i.e., (L, U). Next, if QC_1 is again the first to finish loading containers to Z_1, and it subsequently chooses to cooperate with QC_2 in unloading containers from Z_2, the transition occurs from state 2 to state 4 i.e., $(*, U_j)$. From state 4, the system will move to state 9 i.e. $(*, L)$ if QC_1 leaves the vessel after unloading containers from Z_2 and QC_2 begins loading containers to Z_2. The process terminates when the containers have been loaded to Z_2 and the system reaches absorption at state 13 i.e., $(*, *)$.

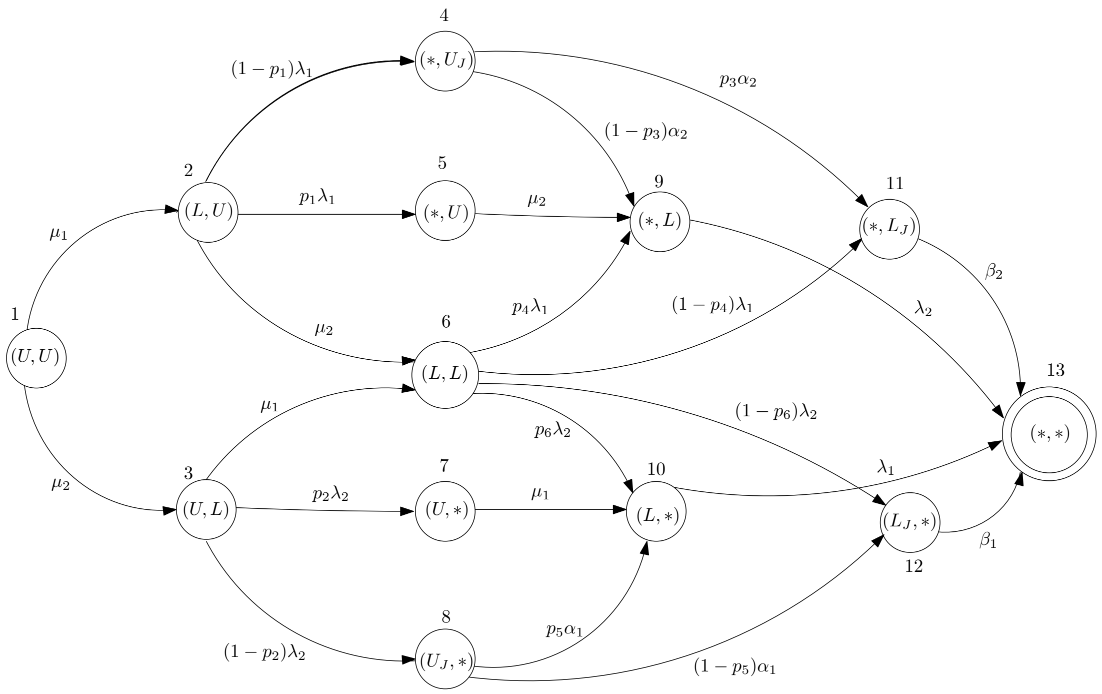

Figure 2: State transition diagram

Sojourn times in a state:

The expected sojourn time in a state is defined as the average duration of time for which the system remains in a particular state before moving to the next state. In our model, the expected sojourn times for each state, denoted by S_j $(j = 1, ..12)$ are shown in Table 2:

Table 2: Expected state sojourn times

State of the CTMC:	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}
Expected Sojourn Time:	$\frac{1}{\mu_1+\mu_2}$	$\frac{1}{\lambda_1+\mu_2}$	$\frac{1}{\lambda_2+\mu_1}$	$\frac{1}{\alpha_2}$	$\frac{1}{\mu_2}$	$\frac{1}{\lambda_1+\lambda_2}$	$\frac{1}{\mu_1}$	$\frac{1}{\alpha_1}$	$\frac{1}{\lambda_2}$	$\frac{1}{\lambda_1}$	$\frac{1}{\beta_2}$	$\frac{1}{\beta_1}$

To explain the expressions obtained in Table 2, consider the expected sojourn time in state 1 i.e., (U, U). In this state, both the QCs are unloading containers from the vessel. Let X_i $(i = 1, 2)$ be the exponentially distributed random variable (with rate μ_i) denoting the time taken to finish unloading by QC_i. The system will leave state 1 only when one of the QCs finishes unloading containers from its zone. It will then move to either state 2 $[(L, U)]$ or state 3 $[(U, L)]$. Thus, the time spent in state 1 will be min (X_1, X_2). Note that min (X_1, X_2) is again an exponentially distributed random variable with parameter $(\mu_1 + \mu_2)$ and hence the expected sojourn time in state 1 is $1/(\mu_1 + \mu_2)$. Similarly, the expected sojourn times for other states can be obtained.

A QC transition rate in the CTMC is in turn dependent on the terminal network parameters such as the QC and the ASC service times (first and the second moments), number of vehicles, and vehicle travel times. The QC throughput rates corresponding to each state in the CTMC are determined by lower level models. A lower level model is a multi-class closed network of queues that models the terminal resources, such as QCs, ASCs, and AGVs. We dedicate AGVs to a QC for transporting containers between quayside and stackside. The QC throughput measure from the closed queueing network provides an estimate of the input parameters to the higher level model. For instance, consider a situation where both QCs are unloading N_i containers each from their respective zones, $i \in \{1, 2\}$. Then the average time to unload N_1 containers from the first zone is $TH_1^{-1}N_1$, where TH_1 is the throughput rate of the first QC from the lower level closed queuing network model. The transition rate from the state (U, U) to the state (L, U) is then given by an exponential parameter, $\mu_1 = TH_1 N_z^{-1}$. In this way our model captures the effect of terminal layout and systems on the vessel sojourn time.

Expected vessel sojourn (throughput) time:
Our objective is to estimate the average time taken to complete the unloading and loading of all containers to the vessel i.e., the time it takes the Markov chain to reach state 13 from state 1. For $j \in \{1, ..., 12\}$; let f_{ij} denote the $(i, j)^{th}$ entry of the fundamental matrix F, i.e., f_{ij} is the average number of visits to state j, starting in state i until absorption. Then f_{1j} gives the average number of visits to state j from state 1 before absorption. S_j is the expected sojourn time in state j. Therefore, $\sum_{j=1}^{12} f_{1j} * S_j$ gives the average time to absorption i.e., the expected vessel sojourn time (see Viswanadham and Narahari [1992] for additional details).

4 Numerical Experiments

Using the stochastic model, we now estimate the vessel sojourn times for balanced parameter settings i.e., an equal number of containers that are unloaded and loaded in both zones of the vessel. The dimensions of the terminal layout and service time parameters for the terminal resources are based on the data obtained from ECT terminal in Rotterdam. The analytical model is implemented in Matlab R2011 software. The two variations of the transition rate input values considered in this study are as follows:

(a) QCs are considered to be the bottleneck resource. In this case, we do not consider the effect of stackside and vehicle path service times on the expected vessel sojourn time. The transition rates for the higher level CTMC model (namely μ_i, λ_i, α_i and β_i for $i = 1, 2$) are obtained directly from the QC service times. The QC service time (for both unloading and loading one container) is assumed to be 100.8 seconds (35 cycles per hour) and 50.4 seconds (70 joint cycles per hour) for individual and joint operations respectively.

(b) The parameters for the higher level CTMC model are estimated by taking into account all the three terminal processes (quayside, stackside and vehicle transport). The service time for QCs is same as in case (a). The average service time of the ASC is 80 seconds each for both unloading and loading the container respectively. The AGV travel time from quayside to stackside and from stackside to quayside are 100 seconds and 140 seconds respectively.

From Table 3, we see that the terminal layout significantly affects the expected vessel sojourn time (in hours) for all 12 cases. The vessel sojourn time is about 10%-18% higher by considering the effect of terminal layout on the QC throughput. As expected, we also see that the expected sojourn time decreases when the QCs cooperate for unloading and loading operations.

Table 3: Expected vessel sojourn time (in hours) under different models

Scenario	**Probability of cooperation (P_i)**	**Zone Z_i ($i = 1, 2$)**		**Analytical model**	
		N_u^i	N_l^i	**(a)**	**(b)**
1	0	500	500	39.3	43.9
2	0.5	500	500	34.8	40.1
3	1	500	500	30.4	36.4
4	0	700	700	53.9	61.5
5	0.5	700	700	47.8	56.3
6	1	700	700	41.7	51.0
7	0	1000	1000	78.6	87.9
8	0.5	1000	1000	69.5	80.3
9	1	1000	1000	60.5	72.8
10	0	1500	1500	119.6	132.2
11	0.5	1500	1500	105.7	120.8
12	1	1500	1500	91.8	109.5

5 Conclusions

In this paper, we present a two-level stochastic model to estimate the expected sojourn time of a vessel. In the higher level model, a CTMC with single absorbing state is considered to model the QC operations on the vessel. The lower level model integrates the quayside, stackside and vehicle transport processes in a closed queuing network. The model is extremely useful to analyze the effect of several terminal design and service parameters on vessel sojourn times. For instance, we can analyze the effect of vehicle travel times, and QC and SC service time variability on vessel sojourn times.

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